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Multiaxial fatigue life prediction of rubber-like materials using the continuum damage mechanics approach

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Abstract

The purpose of this study is to propose a fatigue criterion based on the continuum damage mechanics (CDM) theory to predict the fatigue life of rubber-like materials. The fatigue life of a styrene-butadiene rubber was investigated under tension and twist loadings. Using a generalized Ogden strain energy density function, a CDM model was derived in order to express the fatigue life as a function of an equivalent stress. Since the obtained model was unable to fit all the experimental results, a modified version, which shows a fair agreement with our data, is finally proposed.

Keywords: Rubber-like materials; fatigue life prediction; continuum damage mechanics; hyperelasticity

1. Introduction

Rubber components of mechanical structures could be generally subjected to mechanical loadings that could lead after a certain time of using to progressive damage. Under cyclic loadings, nucleation of micro-defects or micro-cracks is generally observed, their propagation and coalescence leading to complete failure. Therefore, predicting fatigue life under multiaxial loadings could be required when designing rubber components of mechanical structures.

Despite their large area of industrial applications, it should be recognized that many aspects of the fatigue failure of rubber-like materials are still poorly understood. As for metals, two approaches can be used to study the fatigue life of rubber-like materials, under simple or complex loadings. The first approach consists on predicting crack nucleation life using criteria based on stress and/or strain components and coupled with a damage evolution rule (the most used being the Miner rule). This approach postulates that for a given material, it exists an intrinsic fatigue life determined by a criterion generally defined in terms of stresses and/or strains. The relevance of such criteria is closely related to their capability to characterize the material fatigue life regardless of the specimen geometry or the kind of loading. The second one is based on continuum damage mechanics (CDM), in which a damage parameter is computed cycle by cycle and therefore allows the prediction of the fatigue failure of the material.

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When dealing with the crack nucleation, the fatigue life is generally taken as the number of cycles required to create a crack of a given size. It considers that the history of stresses and/or strains at a point of the body allows the determination of the rubber intrinsic life. This approach originally based on the work of Wöhler [1] for steels, was extended to rubber-like materials by Cadwell et al. [2]. It is usually observed that in rubber-like materials the cracks initiate on a plane perpendicular to the maximum tensile strain. So, the maximum principal strain is usually taken as a unified parameter to describe fatigue life. Cadwell et al. [2] and Fielding [3] highlighted that the fatigue life of a natural rubber (NR), under axial and shear tests, could be improved by increasing the minimum stretch. The fatigue life under simple and equibiaxial tension tests was investigated, more recently, by Roberts and Benzies [4]. It was observed that the fatigue life was longer in simple tension than in equibiaxial tension tests when plotted against the maximum strain ratio.

For some authors, criteria based on strains fail because they could not account for the hydrostatic pressure since the rubber-like materials are generally considered as incompressible. So, criteria based on stresses were also proposed. For example, the maximum principal Cauchy stress was used by André et al. [5] and Saintier et al. [6]. In general, it was observed that when the minimum stress is strictly positive the fatigue life increases. The principal Cauchy stress was proposed to be an appropriate criterion to describe multiaxial fatigue damage. It was showed by André et al. [5] that the cracks orientation is perpendicular to the direction of the highest principal Cauchy stress.

Strain energy density (SED) is an evident loading parameter to be used since this scalar quantity can be computed whatever the loading, even multiaxial. But Roberts and Benzies [4] demonstrate that the SED do not lead to a single Wöhler curve for uniaxial and equibiaxial tension tests. Moreover, this scalar quantity could not provide a prediction of the orientation of the nucleated crack. In contradiction with these latest observations, Abraham et al. [7] showed that the SED is a better predictor than strain- or stress-based predictors.

Mars [8], postulating that only a certain part of the SED is available for flaw nucleation, introduced the crack energy density (CED) concept. Under the infinitesimal strain framework, this scalar quantity is defined as the dot product of the Cauchy traction vector by the increment of strain. The material critical plane depicted by its normal which maximizes the CED at a given point is supposed to be the most probable plane where crack can occur for a given loading. Mars [8] extended the previous expression to finite strain framework but the proposed formulation is questionable since the definition of the normal vector was not completely clarified [9,10]. Another new way to derive fatigue criterion is based upon the Eshelby theory of configurational mechanics. Even if this framework was essentially dedicated to investigate fracture mechanics problems, few studies concerning fatigue were achieved. For example, Verron and Andriyana [11] proposed to use the Eshelby tensor as a candidate to describe fatigue crack nucleation in rubber-like materials. Their results show that this concept could be a promising way to treat fatigue problems of rubber.

The fatigue life prediction can be therefore approached by using the above mentioned mechanical quantities but they require to be completed by a damage rule in order to take into account the effects of variable loadings for instance. Another way is to calculate the damage accumulation cycle by cycle using the CDM concept coupled with a fatigue parameter [12]. The main assumption is that, due to the microcracks nucleated from cumulative damage process, the net cross section contributing to the load transfer in a specimen or in a structure decreases with damage. The CDM theory was recently applied to NR by Wang et al. [13]. The damage evolution was derived in the context of CDM developed with a first-order Ogden SED. It was used to describe the fatigue life as a function of the strain amplitude considering only tensile cyclic loading under constant stretch amplitude.

The purpose of this work is first to adapt the approach introduced by Wang et al. [13] by developing it in the case of a generalized Ogden SED function with extension to multiaxial loading. That will lead us to extract a new fatigue parameter which dimension is a stress and called the equivalent stress, depending both on the loading path and on the constitutive law parameters. To check the validity of the proposed criterion which requires two parameters to be identified, fatigue experiments on a styrene-butadiene rubber (SBR) under tension and torsion loadings were achieved. The model parameters were then fitted using the tension tests. Since the results show that predicted values for torsion are clearly underestimated, a modified expression of the equivalent stress parameter is finally proposed which clearly improves the prediction capability of the model.

2. Experimental

2.1. Material and specimen

The selected material is a SBR filled with a 34 phr carbon black and was supplied by Trelleborg. Tab. 1 gives the material contents.

Table 1. Formulation of SBR material (PHR - parts per hundred rubber)

Zinc oxide	Oil	Carbon black	Sulphur	Stearic acid	Antioxidant	Accelerators
10	0	34	3	3	5	4.3

The main sample geometric features are presented in Fig. 1. In order to localize the crack nucleation in the center, a significant radius of curvature R_c was imposed to the specimen.

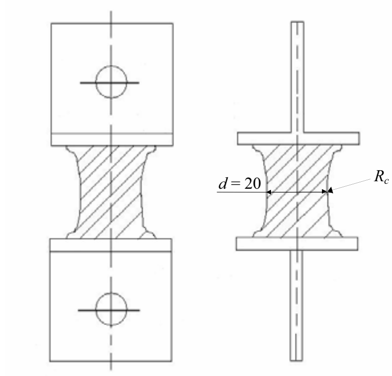

Fig. 1. Specimen geometry for tension and torsion tests (dimensions in mm).

Fatigue tests, under constant displacement (tension) or angle (torsion) amplitude control, were run on a traction-torsion Instron-8872 servo hydraulic testing device, at room temperature under frequencies laying from 3 up to 5 Hz. The loading was based upon a sine wave function of the displacement or of the angle. Current displacement and force data as function of the time were recorded in a computer. The amplitudes of the loading were selected in order to cover a wide range of fatigue lives (from 10^2 to 10^6 cycles).

2.2. Constitutive law

Since the fatigue life model requires the constitutive law of the material to be determined, monotonic (tension and twist) loading tests were also achieved. The obtained experimental data were fitted to an Ogden SED function written as [14]:

$$W = \sum_{i=1}^{n} \mu_i/\alpha_i \left(\lambda_1^{\alpha_i} + \lambda_2^{\alpha_i} + \lambda_3^{\alpha_i} - 3\right) + 9/2\,K\left(J^{1/3} - 1\right)^2 \quad (1)$$

where μ_i and α_i $1 \le i \le n$ are temperature-dependent material parameters, K is the bulk modulus and $J = \lambda_1\lambda_2\lambda_3$ is the Jacobian, equal to 1 for incompressible material.

It is worth noticing that the so-called Mullins effect (stress-softening after one cyclic loading) was cancelled to determine the constitutive law parameters. To this end, each sample was loaded until the maximum stretch amplitude encountered by the material during the fatigue loading and unloaded. It was loaded again and the obtained response was used to fit the Ogden model. A second-order Ogden model was found sufficiently flexible to describe both the mechanical responses in tension and torsion. The optimized values we have obtained are μ_1=0.03 MPa, μ_2=4.03 MPa, $\alpha_1=11\times10^3$ and $\alpha_2=10\times10^{-4}$.

2.3. Experimental fatigue tests

The fatigue tests were conducted under constant displacement or angle amplitude with a zero minimum value (i.e. the load ratio R was zero). Generally, fatigue life of rubber-like materials is taken as the number of cycles required to cause the appearance of a macro-crack (conventionally 1 mm length) or to decrease the stiffness of the specimen by a given amount. In this study, the number of cycles to failure was defined as being as the cycle number corresponding to a visible crack length of 1 mm. The crack on the surface is visually detected by using a magnifying glass associated with a local lighting.
In Table 2 are summarized the loading conditions which were investigated. Are also reported in the table the corresponding number of cycles to failure.

Table 2. Fatigue life results

Loading	Frequency f (Hz)	Maximum displacement δ_{max} (mm)	Maximum angle θ_{max} (°)	Number of cycles N_f
Uniaxial tension UT	5	5.6	—	3 000 000
UT	5	7.0	—	500 000
UT	5	8.4	—	160 000
UT	5	9.8	—	24 000
UT	5	11.2	—	19 200
UT	5	12.6	—	12 564
UT	5	14.0	—	6 000
UT	5	15.4	—	4 300
UT	5	16.8	—	3 050
UT	5	18.2	—	2 200
UT	5	19.6	—	1 800
UT	5	21.0	—	1 350
UT	5	22.4	—	1 050
UT	5	23.8	—	900
UT	5	25.2	—	800
UT	5	26.6	—	650
UT	5	28.0	—	560
Pure torsion PT	3	—	100	1 100 000
PT	3	—	105	1 000 000
PT	3	—	110	680 000
PT	3	—	115	400 000
PT	3	—	120	280 000
PT	3	—	125	200 000
PT	3	—	130	130 000
PT	3	—	135	100 000
PT	3	—	140	79 000
PT	3	—	145	68 000

As an example, Fig. 2 shows the evolution of the maximum load as function of the number of cycles under both uniaxial tension (UT) and pure torsion (PT). Under constant displacement amplitude, the stress level highly decreases after a short time, and then it remains quite constant until the onset of crack nucleation. This is certainly the consequence of self-heating generally observed for rubber-like materials. In the case of torsion the decrease is more continuous until the failure of the specimen.

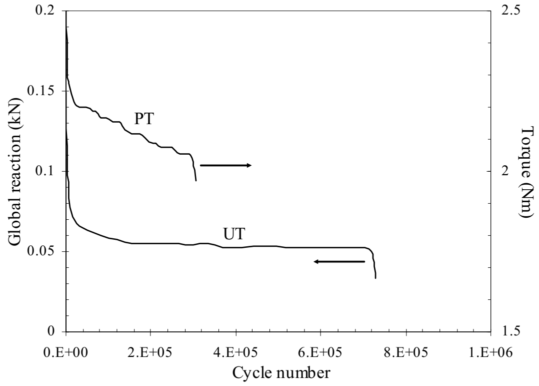

Fig. 2. Maximum load as a function of the cycle number under uniaxial tension (UT) and pure torsion (PT) conditions.

3. CDM model

As above mentioned, a CDM model based upon the effective stress concept [12] is developed. The rubber material is assumed as an isotropic, homogeneous, incompressible and hyperelastic body. The initiation, growth and coalescence of micro-defects lead to the degradation of material properties. The introduction of an internal field variable, associated to damage evolution in the CDM model allows capturing the material properties degradation process. As a result of such a postulate, the effective stress tensor $\tilde{\boldsymbol{\sigma}}$ is related to the Cauchy stress tensor $\boldsymbol{\sigma}$ using the effective stress concept [12]:

$$\boldsymbol{\sigma} = (1 - D)\tilde{\boldsymbol{\sigma}} \quad (2)$$

in which D is a scalar defining the isotropic damage evolution. The damage variable D is zero in the initial state (virgin material) and 1 for fracture (when a macroscopic crack is initiated).

Eq. (2) can be written, under the framework of finite strain and Lagrangian description, when using the second Piola Kirchhoff stress tensor $\mathbf{S}$, or in the same manner using its components S_i in the principal directions, as follows:

$$S_i = (1 - D)\tilde{S}_i \quad (3)$$

where S_i are the current principal stresses and $\tilde{S}_i$ are the effective principal stresses.
The isotropic damage evolution is given by [12]:

$$\dot{D} = -\partial\varphi^* / \partial y \quad (4)$$

where φ^* is the dissipation potential and y is the damage strain energy release rate. The dissipation is expressed by the following power-law form [12]:

$$\varphi^* = A\left(a+1\right)^{-1}\left(-yA^{-1}\right)^{a+1} \tag{5}$$

where a and A are damage parameters to be identified using experimental data.

As the SED is a function of the principle stretches, i.e. $W = W\left(\lambda_i, D\right)$, the damage strain energy release rate y can be written in the following form:

$$-y = \partial W/\partial D = \sum_{i=1}^{3}\left(\partial W/\partial\lambda_i\right)\left(\partial\lambda_i/\partial D\right) \tag{6}$$

Differentiating of $\tilde{S}_i$ in Eq. (3) with respect to the damage variable D leads to:

$$\partial\tilde{S}_i/\partial D = S_i/\left(1-D\right)^2 = \left(\partial\tilde{S}_i/\partial\lambda_i\right)\left(\partial\lambda_i/\partial D\right) \tag{7}$$

Eqs. (3) and (7) lead to:

$$\partial\lambda_i/\partial D = \tilde{S}_i\left(\left(1-D\right)\partial\tilde{S}_i/\partial\lambda_i\right)^{-1} \tag{8}$$

To go further, it is necessary to choose a SED function. In this work, a generalized Ogden constitutive law was selected (see Eq. (1)).

Using Eqs. (1) and (6), the damage strain energy release rate y could be then derived:

$$-y = S_{eq}\left(1-D\right)^{-1} \tag{9}$$

S_{eq} is the fatigue parameter which can be seen as an equivalent stress able to represent any multiaxial loading and given by :

$$S_{eq} = \sum_{i=1}^{3}\left(\sum_{j=1}^{n}\mu_j\lambda_i^{\alpha_j-2} + p\lambda_i^{-2}\right)\left(\sum_{j=1}^{n}\mu_j\lambda_i^{\alpha_j-1} + p\lambda_i^{-1}\right)\left(\sum_{j=1}^{n}\mu_j\left(\alpha_j-2\right)\lambda_i^{\alpha_j-3} + \partial p/\partial\lambda_i\,\lambda_i^{-2} - 2p\lambda_i^{-3}\right)^{-1} \tag{10}$$

where p is the hydrostatic pressure which is an unknown quantity introduced in the model formulation since the material is assumed incompressible. To determine the hydrostatic pressure expression, the boundary conditions must be used.

The damage kinetics is then expressed using Eqs. (4), (5) and (9):

$$\dot{D} = \left(S_{eq}A^{-1}\left(1-D\right)^{-1}\right)^{a} \tag{11}$$

Under a cyclic loading condition, the damage will accumulate with the number of cycles and the damage evolution will depend on the strain amplitude. The time rate change of damage variable $\dot{D}$ can be represented in terms of the evolution of D with respect to the number of cycles. Based on this consideration the fatigue damage evolution per cycle is then expressed as:

$$\partial D/\partial N = \left(S_{eq}A^{-1}\left(1-D\right)^{-1}\right)^{a} \tag{12}$$

Reminding that the damage variable D is zero for the virgin material then the damage value at any cycle can be determined by integrating Eq. (12), which gives:

$$D = 1 - \left[1 - (1+a)\left(S_{eq} A^{-1}\right)^{a} N\right]^{1/(1+a)} \quad (13)$$

Thus, the fatigue failure is assumed to occur when the damage value reaches its maximum value $D_f = 1$ corresponding to the number of cycles N_f. The fatigue life can be therefore easily deduced from Eq. (13) and expressed in the following power-law form:

$$N_f = A^{a} (1+a)^{-1} S_{eq}^{-a} \quad (14)$$

The damage parameters can be easily identified by using a square least method.

4. Results and discussion

4.1. Validity of the model

In this subsection, we propose to check the validity of the above described criterion by comparing predicted values to our experimental results. The stress and strain tensors components are extracted in the region of the specimen where they are maximum by using a FE analysis for tested specimens. A square least method allows the two parameters a and A to be identified by using experimental data obtained under UT tests. The optimized values we have obtained are A = 13.12 MPa and a = 2.68. The fatigue life is then plotted in Fig. 3 as a function of the equivalent stress defined by Eq. (10). It can be clearly observed that using such a defined equivalent stress could not allow unifying the experimental data corresponding to UT and PT tests. We can clearly observe that the predicted values in torsion highly underestimate the corresponding experimental data. Such a bad result encourages us to modify the model in order to get best predicted values.

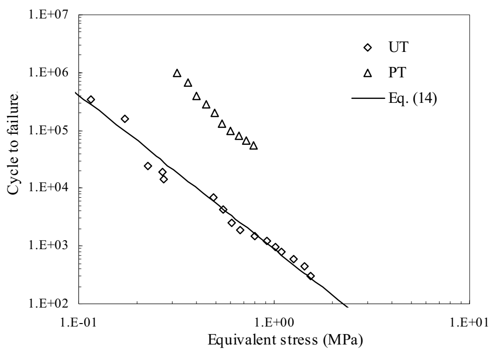

Fig. 3. Fatigue life as a function of the equivalent stress given by Eq. (10).

4.2. Model modification

Since the model was developed according to a formalism based upon the second thermodynamic principle, the major part of the mathematical developments was kept. We only postulate that the damage due to the maximum principal stress (or maximum principal stretch) is predominant in the fatigue process (which is generally experimentally confirmed) so that the damage strain energy release rate given by Eq. (6) could be simplified and approximated as follows:

$$-y = \partial W/\partial D \approx \left(\partial W/\partial \lambda_1\right)\left(\partial \lambda_1/\partial D\right) \quad (15)$$

That allows to define a modified expression of the fatigue parameter (i.e. the equivalent stress) which is derived from Eq (10) using the approximation expressed in Eq. (15). That leads to a new fatigue predictor written in the following form:

$$S_{eq}^{corr.} = \left(\sum_{j=1}^{n} \mu_j \lambda_1^{\alpha_j - 2} + p\lambda_1^{-2}\right)\left(\sum_{j=1}^{n} \mu_j \lambda_1^{\alpha_j - 1} + p\lambda_1^{-1}\right)\left(\sum_{j=1}^{n} \mu_j \left(\alpha_j - 2\right)\lambda_1^{\alpha_j - 3} + \partial p/\partial \lambda_1\, \lambda_1^{-2} - 2p\lambda_1^{-3}\right)^{-1} \quad (16)$$

The suffix "*corr.*" in the equivalent stress reminds that the expression is based upon an approximation of the damage strain energy release rate.

Keeping the same procedure that the above mentioned one to identify the parameters a and A of the model, Fig. 4 clearly shows that the proposed modification in the expression of the equivalent stress leads to a significant improvement of the model which capability to predict the experimental results under torsion loading is fairly highlighted.

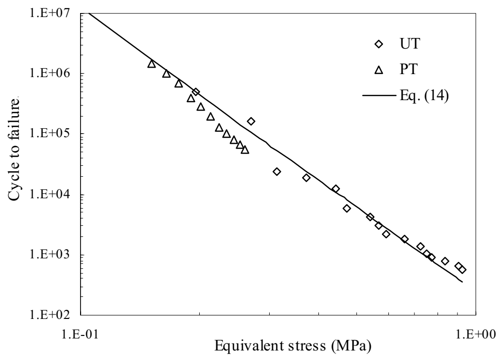

Fig. 4. Fatigue life as a function of the equivalent stress given by Eq. (16).

This is also confirmed in Fig. 5 when plotting the data in a diagram with experimental fatigue life as ordinate and corresponding predicted values as abscissa. All the data are closer to the median line showing the capability of the so defined equivalent stress to be taken as a fatigue parameter when dealing with fatigue of rubber-like materials.

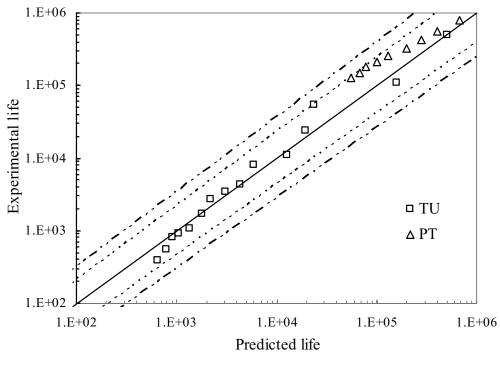

Fig. 5. Comparison between predicted and experimental fatigue life results.

5. Conclusion

In the present contribution, a model for fatigue life prediction of rubber-like materials under multiaxial loading conditions was proposed. The model is based on the CDM framework and its extension to rubber developed by Wang et al. [13]. The formulation was first extended to a generalized Ogden SED function. The predictive capability of the model was checked on fatigue tests in UT and PT achieved on the SBR material under constant amplitude loading conditions. Taking the UT experimental data as a reference to identify the model parameters, it was first found that the model is unable to predict the data obtained in torsion. A modified version was then proposed, taking only into account the effects of the maximum principal strain. In this case, it was found that the model reproduces in a satisfactory manner the experimental fatigue life of the studied material.

In addition to the (usual) material constants of the constitutive response, only two additional damage parameters are needed to predict the fatigue life of rubber-like materials, determined by a simple linear regression method on UT tests under fatigue loading. In order to improve the modelling there is a need to incorporate the effect of *R*-ratio. Moreover, it is well-known that some rubber-like materials are strain rate-dependent. It could be then important to take into account the viscous effects, responsible of self-heating. The temperature dependence of the constitutive law could be also a constraint that requires to be accounted for.

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